GCSE Maths – Algebra

Solving Quadratic Inequalities (Higher Only)

Notes

WORKSHEET

Solving Quadratic Inequalities (Higher Only)

Quadratic Inequalities

Quadratic inequalities are inequalities which involve the term x^2. These inequalities can be solved by **sketching a quadratic graph.**

Example: Solve the inequality $x^2 + 5 < 14$. Present your answer on a number line.

1. **Rearrange** the inequality in the form of a quadratic equation $(ax^2 + bx + c < 0)$. The right-hand side should be equal to 0.

$$x^2 + 5 < 14$$

Subtract 14 *from both sides of the equation:*

$$x^2 - 9 < 0$$

2. **Find the** x **–intercept** by **factorising** the quadratic equation in the inequality.

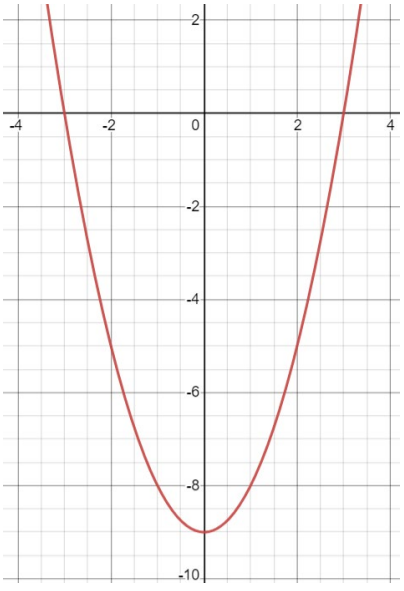
$$x^2 - 9 = 0$$
$$(x + 3)(x - 3) = 0$$

$$x + 3 = 0 \quad \textit{or} \quad x - 3 = 0$$
$$x = -3 \qquad\qquad x = 3$$

The coordinates of the x –intercepts are $(-3, 0)$ and $(3, 0)$.

3. Using the **coordinates** of the x –intercept and the y –intercept, **sketch a quadratic graph** equivalent to the quadratic inequality.

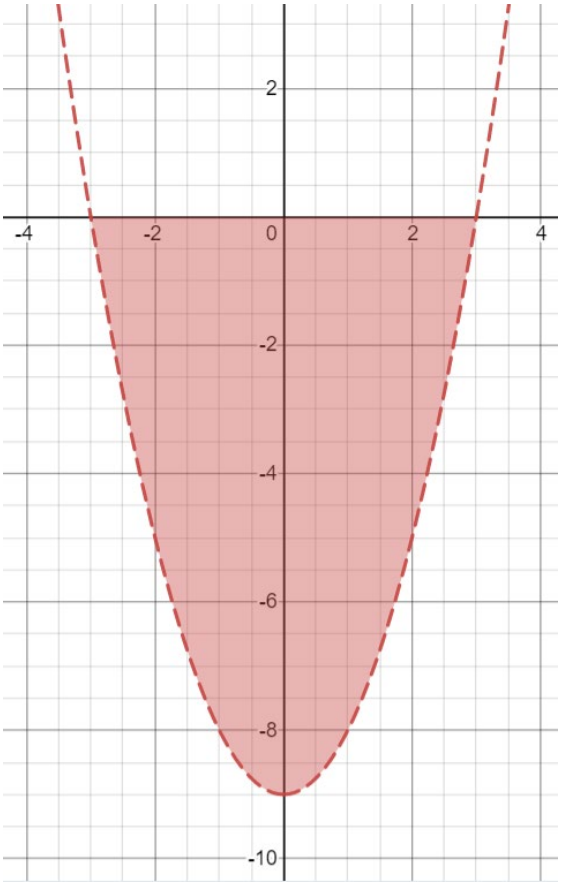
For the equation $y = x^2 - 9$, *the y-intercept is* $(0, -9)$. *From step 2, the x-intercepts are* $(-3, 0)$ *and* $(3, 0)$.



Remember, inequalities that use $<$ *or* $>$ *symbols are plotted with a dashed line to show that the line is not included in the region.*

4. **Identify the required region** which satisfies the quadratic inequality.

   For $x^2 - 9 < 0$, the region satisfied should be the area where the quadratic is less than 0. In this case, the area should be the region below the x −axis, as shaded in the graph.

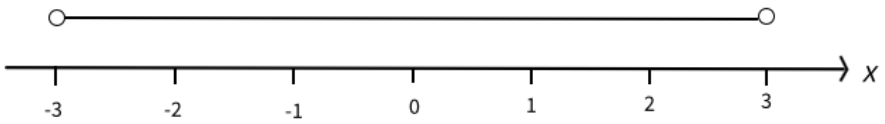


5. **Give the answer** of the region satisfied **in the form of an inequality**. If the shaded region is only one region, the answer should be in one inequality.

   The region shaded is the values of x where $-\mathbf{3} < \boldsymbol{x} < \mathbf{3}$. Therefore, these are the values of x that satisfy the inequality $x^2 + 5 < 14$.

6. Represent the answer on a **number line**.

   The solution $-\mathbf{3} < \boldsymbol{x} < \mathbf{3}$ represented on a number line is:



In certain cases, the region which satisfies the quadratic inequality will be the union of two separate regions. In this case, the answer will involve two inequalities.

Example: Solve the inequality $x^2 - 3 \geq 6x + 4$. Present your answer on a number line.

1. **Rearrange** the inequality in the form of a quadratic equation $(ax^2 + bx + c \geq 0)$.
   The right-hand side should be equal to 0.

$$x^2 - 3 \geq 6x + 4$$
$$x^2 - 6x - 3 \geq 4$$
$$x^2 - 6x - 7 \geq 0$$

2. **Find the x –intercept** by **factorising** the quadratic equation equivalent to the inequality.

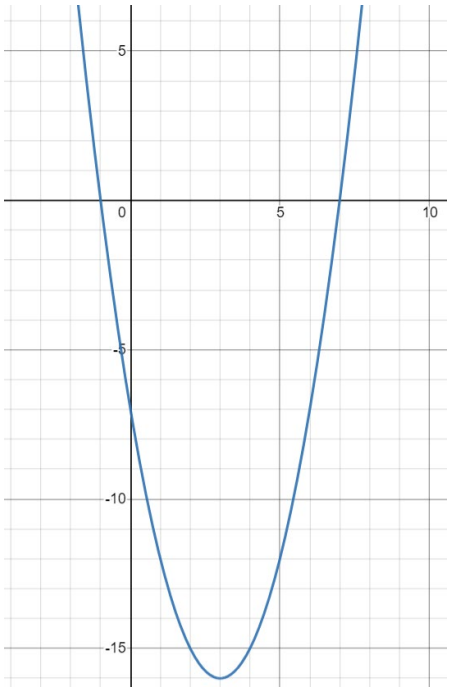
$$x^2 - 6x - 7 = 0$$
$$(x + 1)(x - 7) = 0$$

$x + 1 = 0$ *or* $x - 7 = 0$
$x = -1$ $x = 7$

The coordinates of the x –intercepts are $(-1, 0)$ and $(7, 0)$.

3. Using the **coordinates** of the x –intercept and the y –intercept, **sketch a quadratic graph** equivalent to the quadratic inequality.

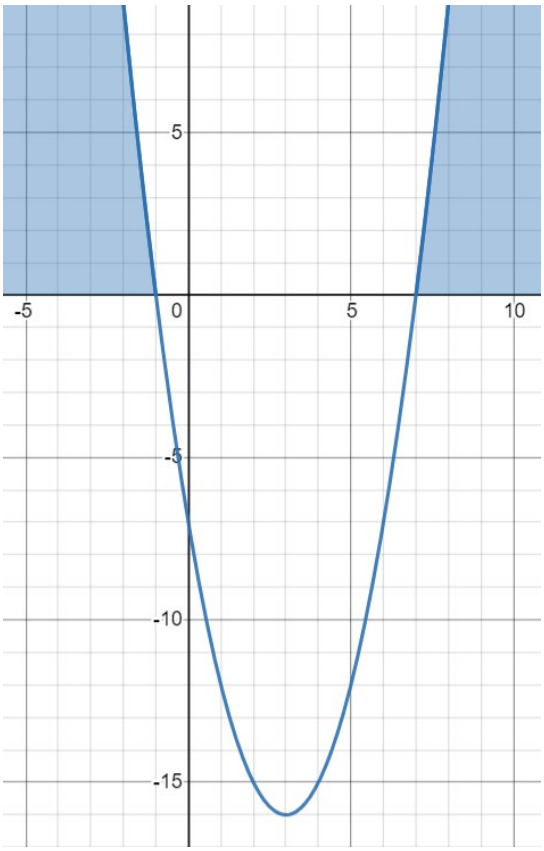
For the equation $y = x^2 - 6x - 7$*, the y-intercept is* $(0, -7)$*. From step 2, the x-intercepts are* $(-1, 0)$ *and* $(7, 0)$.



Remember, inequalities that use $\leq$ *or* $\geq$ *symbols are plotted with a solid line to show that the line is included in the region.*

4. **Identify the required region** which satisfies the quadratic inequality.

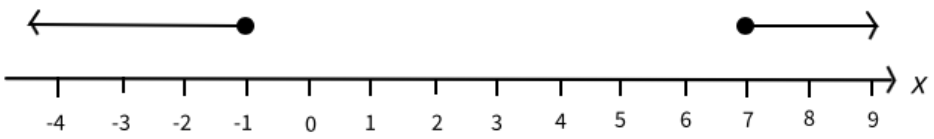
   For $x^2 - 6x - 7 \geq 0$, the wanted region should be the area where the quadratic is greater or equal than 0. In this case, the area should be above the x −axis, as shaded in the graph.



5. **Write the answer** of the required region **in the form of an inequality**. Since the shaded regions consist of two areas, the answer should be in **two inequalities**.

   The required region is $\boldsymbol{x \leq -1}$ or $\boldsymbol{x \geq 7}$.

6. Represent the answer on a **number line**.



Solving Quadratic Inequalities – Practice Questions (Higher Only)

1. Solve the following inequalities and present your answer in a number line.

   a) $3x^2 + 2x < 14 + 2x^2 - 3x$

   b) $x^2 + 5 > 30$

   c) $-2x^2 + 4 \geq x^2 + 9x - 8$

   d) $x^2 + 12 \geq -7x + 2$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.